Journal of Hazardous Materials 190 (2011) 51–59

Contents lists available at ScienceDirect

Journal of Hazardous Materials

journal homepage: www.elsevier.com/locate/jhazmat

Pretreatment of wastewater: Optimal coagulant selection using Partial Order Scaling Analysis (POSA)

Eran Tzfati [a], Maya Sein [a], Angelika Rubinov [a], Adi Raveh [b], Amos Bick [a,c,*]

[a] Department of Chemical Engineering, Shenkar College of Engineering and Design, 12 Anna Frank street, Ramat Gan 52526, Israel
[b] The School of Business Administration, The Hebrew University, Jerusalem, Israel
[c] Department of Industrial Engineering and Management, Jerusalem College of Technology, 21 Havaad Haleumi St., Jerusalem 91160, Israel

ARTICLE INFO

Article history:
Received 14 September 2010
Received in revised form 2 January 2011
Accepted 8 February 2011
Available online 15 February 2011

Keywords:
Coagulation
Jar test
Management modeling
Multi-criteria decision
Partial order analysis
Treatment selection

ABSTRACT

Jar-test is a well-known tool for chemical selection for physical–chemical wastewater treatment. Jar test results show the treatment efficiency in terms of suspended matter and organic matter removal. However, in spite of having all these results, coagulant selection is not an easy task because one coagulant can remove efficiently the suspended solids but at the same time increase the conductivity. This makes the final selection of coagulants very dependent on the relative importance assigned to each measured parameter. In this paper, the use of Partial Order Scaling Analysis (POSA) and multi-criteria decision analysis is proposed to help the selection of the coagulant and its concentration in a sequencing batch reactor (SBR). Therefore, starting from the parameters fixed by the jar-test results, these techniques will allow to weight these parameters, according to the judgments of wastewater experts, and to establish priorities among coagulants. An evaluation of two commonly used coagulation/flocculation aids (Alum and Ferric Chloride) was conducted and based on jar tests and POSA model, Ferric Chloride (100 ppm) was the best choice. The results obtained show that POSA and multi-criteria techniques are useful tools to select the optimal chemicals for the physical-technical treatment.

1. Introduction

One of the most commonly used methods for the removal of suspended solids in wastewater is the addition of coagulant and flocculation aids, such as Alum, Ferric Chloride, and long chain polymers [1]. Coagulation, flocculation and clarification, followed by filtration, are the key steps in conventional wastewater treatment systems. This is a well-proven technology for the significant removal of color and particulate matter including protozoa (e.g. *Cryptosporidium* oocysts and *Giardia* cysts), viruses, bacteria, and other micro-organisms. Iron, manganese, tastes and odors may also be removed from the water by these processes [2,3].

The treatment has several distinct stages: a coagulant is added to neutralize the natural electrical charges on the colloidal particles that prevent them from agglomerating, and is rapidly mixed into the water to be treated. The processed water will then enter a flocculation chamber and a gentle mixing during this stage allows particles to agglomerate and form settleable flocs. Clarification usually follows the flocculation process and involves sedimentation or settling, which allows the formed flocs to be separated for subsequent removal as sludge. Clarification is then followed by filtration which provides a second, polishing step for particulates that were not removed during the clarification step [4].

Nearly every water treatment plant uses Aluminum-based coagulants [e.g. Aluminum-sulphate (Alum) or Poly-Aluminum Chloride (PACl) or iron-based coagulants (Ferric Chloride or Ferric Sulphate)] [5–13]. Alum has been used for several centuries in water treatment and is probably the most well known and commonly used coagulant. The chemical is prepared by the reaction of certain clays with H_2SO_4 acid and delivered in granular or powder form. This coagulant is acidic in nature and its storage and handling require corrosion-proof tanks, pumps and pipes [7].

Ferric Chloride is highly acidic and the solution contains free hydrochloric acid. The solution is highly corrosive to nearly all normally used metals including all grades of stainless steel and needs to be stored, pumped and conveyed in synthetic corrosion-resistant materials. The chemical is normally supplied as a solution of about 40% strength as $FeCl_3$ with a specific gravity of about 1.4 and a pH of less than 1.0 [8].

The best approach for determining the treatability of a water source and determining the optimum parameters (most effective

* Corresponding author at: Department of Industrial Engineering and Management, Jerusalem College of Technology, 21 Havaad Haleumi St., Jerusalem 91160, Israel. Tel.: +972 35349053; fax: +972 35352242.

E-mail addresses: etzfati@walla.com (E. Tzfati), maya_shine@hotmail.com (M. Sein), angel@mail.shenkar.ac.il (A. Rubinov), msraveh@mscc.huji.ac.il (A. Raveh), amosbick@gmail.com (A. Bick).

0304-3894/$ – see front matter © 2011 Elsevier B.V. All rights reserved.
doi:10.1016/j.jhazmat.2011.02.023

coagulant, required dose rates, pH, and flocculation times) is by using a jar tester [14].

It is always preferable to carry out tests on a number of samples and if possible, under different conditions to establish the most reliable product. Having selected a suitable product, the use of routine jar tests remains necessary for a number of reasons: (i) the nature and quality of the raw water may change, which may affect the coagulant dose, (ii) it is necessary to check that the plant dosage matches the demand established in the laboratory, and (iii) different batches of coagulant may vary and the use of comparative jar tests using some of the original product sample is a useful quality check. The normal procedure when conducting a jar test is; (i) initially to find the best performing coagulant and dose rate, and; (ii) to determine the optimum pH for the chosen coagulant and dose rate. Coagulation/flocculation performance is usually judged on the basis of turbidity, color and organic constituent removal [15–18].

In this work, multi-criteria decision analysis is proposed as a tool for helping in the design of the physical–chemical wastewater treatment (jar test). Multi-criteria is a term that includes a set of concepts, methods and techniques that seek to help individuals or groups to make decisions, which involve several points of view in conflict and multiple stakeholders. All these concepts and methods (Factor Analysis, Principal Component Analysis, Correspondence Analysis, Cluster Analysis, and Multidimensional Scaling) have been largely studied in the operational research literature and are based on distance, coordination and absolute or nominal scaling [19,20].

Partial order analysis is based on hierarchy and product of chain (where a chain is defined as an ordered set in which any two elements are comparable with respect to the given order relation, like the test scores). Partial order as a discipline of discrete mathematics appears to be a promising tool for decision-making particularly in environmental issues [21]. It has been argued that partial order theory may be the most objective way to rank a set of elements [22]. The objectivity lies in the fact that in contrast to other multi-criteria methodologies, there is no need to unify the descriptors using weighting coefficients in any kind of functional relationships. The partial order model implementation is discussed in Section 2.1.2.

In partial order ranking, in contrast to standard multidimensional statistical analysis, neither assumptions about linearity nor any assumptions about distribution properties are made. In this way the partial order ranking can be considered as a non-parametric method. Thus, there is no preference among the descriptors. However, due to the simple mathematics outlined above, it is obvious that the method a priori is rather sensitive to noise, since even minor fluctuations in the descriptor values may lead to non-comparability or reversed ordering. Conventional partial order ranking may appear insufficient to solve problems involving a high number of parameters as the number of parameters may well appear to be prohibitive for developing a robust model [23]. A possible improvement is to apply weights within a step-by step procedure or to use fuzzy partial order concepts [24].

The objective of this study was to define a simple procedure useful in selecting the best coagulant and the relative dose for the treatment of wastewater. The specific aim was to design and optimize coagulation/flocculation process for the treatment of Municipal Solid Waste (MSW) in Hiria (Israel): (i) to investigate a first set of jar tests using different concentrations of Alum and Ferric Chloride for wastewater treatment, (ii) to add economical parameters in order to weight these parameters, according to the judgments of wastewater experts, (iii) to maximize the removal of organic constituents, minimize coagulant dose & cost and optimize the performance of a wastewater system which was equipped with a coagulation/flocculation process, and; (iv) to estimate the application of the Partial Order Scaling Analysis for decision making in water and wastewater technology.

2. Materials and methods

2.1. Management modeling and Partial Order Scaling Analysis (POSA)

2.1.1. General used models for coagulant selection

There is a tremendous need for research and models in the field of wastewater treatment and a great deal of research is required to accurately define the magnitude of adverse effects of the waste generated from various plants that use coagulation technology (especially clinical laboratories and multi-specialty hospitals) [25]. The limitations of using jar tests for determining optimum coagulant doses can be overcome by using models. Concerning the academic literature, there are few economic, statistical and multivariate models for optimal coagulant selection [13]. The technical and the economic studies are based on: (i) the identification of the profiles for various design and operating parameters, and; (ii) computation of the total annual cost (for selected coagulant, capacity and waste characteristics) for diverse pH at different coagulant doses [26].

Concerning statistical models, there are two types for coagulant selection: (i) simulation (linear and multifactor nonlinear), and; (ii) process. Contrary to the multiple regression models, the general linear model can analyze simultaneously more than one dependent variable. The selection of the optimum coagulation conditions is carried out by post hoc analysis using Duncan test. Pos hoc analysis determines if a certain difference between removal efficiencies is actually significant or not [27].

In conventional multifactor experiments, optimization is usually carried out by varying a single factor while keeping all other factors fixed at a specific set of conditions. It is not only time-consuming, but also usually incapable of reaching the true optimum due to ignoring the interactions among variables. On the other hand, the Response Surface Methodology (RSM) has been proposed to determine the influences of individual factors and their interactive influences. The RSM is a statistical technique for designing experiments, building models, evaluating the effects of several factors, and searching optimum conditions for desirable responses. With RSM, the interactions of possible influencing parameters on treatment efficiency can be evaluated with a limited number of planned experiments [28–32].

When process models are used, the data include process inputs (e.g. raw water quality parameters) and process control parameters (e.g. coagulant dose, pH) and the outputs of the process that is being modeled (e.g. treated water quality parameters) [33–36]. Although the utilization of process and models overcomes the limitations of using jar tests for determining the optimal Alum or Ferric Chloride dose, the development of such models is not a trivial task. This is because water treatment processes are governed by complicated, nonlinear relationships [36].

2.1.2. Partial Order Scaling Analysis (POSA)

Multi-criteria analysis is a difficult task and typically involves subjective mutual weight [37]. One possibility to overcome these problems is to apply partial order ranking methodology [38–43]. Partial Order Analysis (POSA) is defined as the empirical determination of the dimensionality of partly ordered scalograms and is based on elementary methods of discrete mathematics [44]. Methods related to partial order theory such as the Hasse Diagram Technique (HDT) are increasingly used in the field of multi-criteria decision support [45–48] and appear as an attractive and simple tool to assess priorities [49–53]. Despite the well-known total rank-

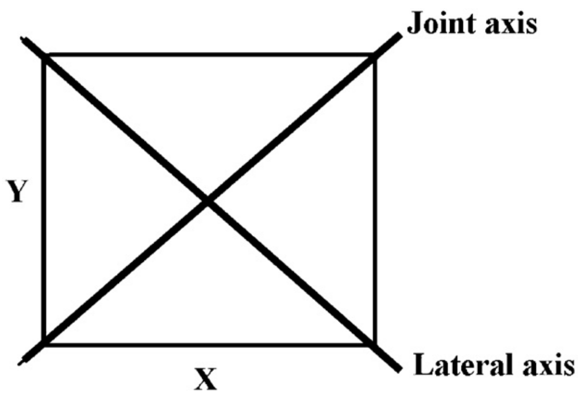

Fig. 1. Joint and lateral axes in POSA [57].

ing strategies, which are scalar methods combining the different criteria values into a global index which always ranks elements in an ordered sequence, the partial order ranking is a vector methodology which recognizes that not all the elements can be directly compared with all the others [54–56].

The following example demonstrates this perception. Consider a group of individuals with a vector of four tests: T1, T2, T3 and T4. An individual receives a score of 1 in T1 if he fails test T1, and a score 2 if he passes that test. Similarly, in each of the other tests: a score of 1 is assigned to an individual for failure and 2 for success. The outcome of administering the four tests to an individual is, of course, a set of four scores (vector) which may be listed in the order of the test-designation. For instance, an individual's outcome may be (1222) if he failed in test T1 and passed T2, T3 and T4, or (2121), if he passed tests T1 and T3 but failed in tests T2 and T4.

Each profile describes a vector that performs better than another if at least one variable is performed better than the other. Thus (2112) is higher than (1111). These two sources are considered comparable. Two sources are not comparable if one variable of the 1st source is higher and another variable of the 2nd source is higher. Thus (2121) and (2112) are not comparable.

In brief, partial order ranking is a simple principle, which a priori includes "$\leq$" as the only mathematical relation, adopts the principles described above and represents the resulting variation among profiles as points in the geometric space shown in Fig. 1. Data records with identical profiles are represented in the POSA space by the same point as there is neither a quantitative nor a qualitative basis for discriminating between them [57]. Profiles differing in the degree of competitiveness are ordered according to their sum of structs as points along the joint axis from bottom-left (i.e., 1111) to top-right (i.e., 2222). Profiles that involve the same degree of competitiveness but differ in the type of behavior (non-comparable) are represented as points spread along the lateral axis that spans from top-left to bottom-right (Fig. 1).

However, because non-comparable profiles can occupy one of several positions along the lateral axis while still preserving an order of increasing competitiveness, POSA arranges the configuration so that the more structs two profiles have in common, the closer their representative points appear in the solution space. For example, the profiles (2211), (2112), (1212) and (1122) would be distributed along the lateral axis because they involve the same degree of competitiveness (i.e., sum of structs equals 6) but different types of behavior (i.e., patterns of scores across the structs). In positioning these profiles, POSA would put (2211) and (1212) in close proximity since they have a common score on the second and third struct, while (2211) and (1122) would be positioned at opposite ends of the lateral axis because they have no structs in common. Profiles (2211) and (1122) are considered to be more dissimilar in the type of occurring behavior than the profiles (2211) and (1212). The partial order creates a diagram for each variable in which the profiles are represented by points in space. The higher the score or category a profile receives on a specific variable, the better its performance.

In general, POSA determines the placement of profiles along the joint and lateral axes through a process in which profiles with the same score on a struct (behavioral variable) are positioned closer together in the solution space than profiles with different scores on that struct. Specifically, each struct of a profile is considered separately, and profiles with the same score on the struct being examined are positioned into a contiguous region of the solution space. Thus, for each struct, POSA attempts to position profiles in such a way that drawing straight lines through the solution space can separate profiles scoring a 1 or 2 on the relevant struct.

2.1.3. Partial Order Scaling Analysis (POSA): implementation

POSA compares individuals in terms of similarities of profiles from particular variables and represents these similarities geometrically as distances in space. It assumes some underlying order to the variables selected and builds the geometric representation around this order; such that the cases that score highest appear in the top right hand corner of the plot and those cases which score lowest on the variables appear towards the bottom left corner of the plot. This procedure is fully described and illustrated [58,59] by academic computer software [60–63]. Several POSA models are in use for academic research (POSAC, HUDAP, and CoPlot). The weak monotonicity coefficient, denoted by μ_{AB} is of special use by POSA software [57]. The formula for weak monotonicity (μ_{AB}) between two tests with ordered ranges, A and B is as follows:

$$\mu_{AB} = \frac{\sum_{q=1}^{N}\sum_{p=1}^{N}(a_p - a_q)(b_p - b_q)}{\sum_{q=11}^{n}\sum_{p=1}^{n}|a_p - a_q||b_p - b_q|} \tag{1}$$

where a_p and a_q are (subject) p's and (subject) q's test scores respectively in one test record, and b_p and b_q are p's and q's test scores in the other test record. The weak monotonicity coefficient μ_{AB} expresses the extent to which the information on one variable increases in a particular direction as the information to the other variable increases, without assuming that the increase is exactly according to a straight line. It varies between −1.00 and +1.00. The weak monotonicity μ_{AB} = +1.00 implies a perfect monotone trend in a positive direction and μ_{AB} = −1.00 implies a perfect monotone trend in a negative or descending direction [57].

The goodness-of-fit of the technique can also be assessed by using the coefficient of alienation Θ and is embedded in CoPlot software [64].

$$\Theta = \sqrt{1 - \mu_{AB}^2} \tag{2}$$

CoPlot's output is a visual display of its findings and it is based on two graphs that are superimposed on each other [65]. The first graph maps the n observations into a two-dimensional space. This mapping, if it succeeds, conserves distance: observations that are close to each other in n dimensions and are also close in two dimensions, and vice versa. The second graph consists of n arrows, representing the variables, and shows the direction of the gradient along each one. Each variable vector is chosen from a least-square regression so that the correlation between the actual values for the variable (from the original Y matrix) and the distances from the projections of each observation onto the vector is maximized (i.e. CoPlot chooses the j th vector so that the correlation of the values of variable j and the projections of each observation on the j th vector is maximized).

The aim of POSA is to present the data in two dimensions, preserving the partial order as well as possible in a mapping diagram in order to define clusters. It is clear that for, empirical data, this

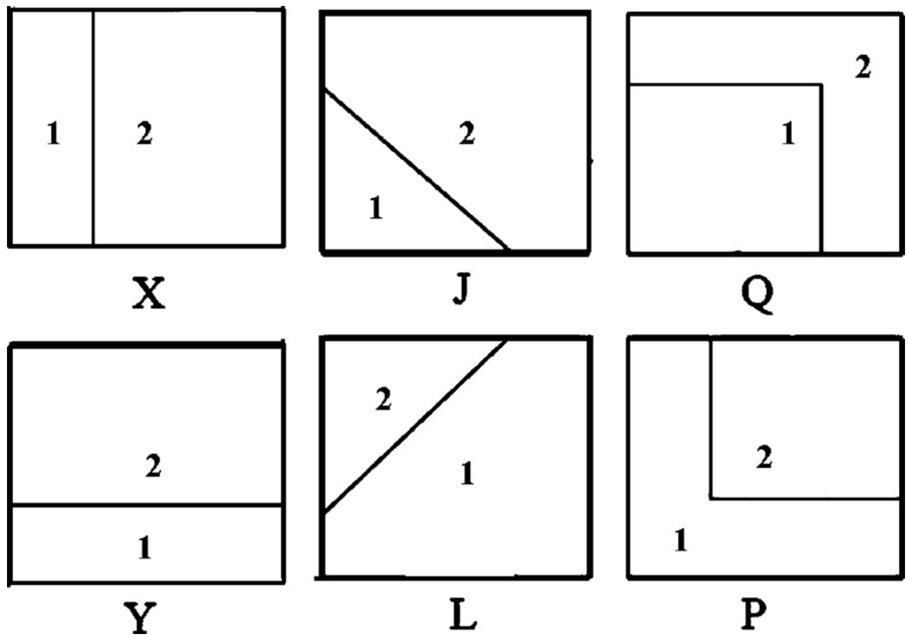

Fig. 2. Partitioning options representing the quantitative structure of the profiles [57].

procedure introduces some error in the structured map. So, a coefficient which measures the goodness of fit must be defined. POSAC gives such coefficient (CORREP):

$$\mathrm{CORREP} = \frac{N_C^* - N_I^*}{N_C^* + N_I^*} \tag{3}$$

where N_C^* and N_I^* are the numbers of subject-pairs correctly represented, and, respectably, incorrectly represented by POSAC. Concerning a specific test (or variable) each profile obtains 1 if he fails the test and 2 if he passes that test. Hence, the Ratio of Subject Pairs Correctly Represented to the total number of subject pairs (RSPCR) is actually [57]:

$$\mathrm{RSPCR} = \frac{1 + \mathrm{CORREP}}{2} \tag{4}$$

RSPCR coefficient ranges between 0.00 and 1.00, with a score of 1.00 indicating that all records with a particular struct score may be partitioned into a region of the solution space without exception. In general, a coefficient above 0.80 is regarded as indicating that scores on a particular struct vary systematically across the partial order and that, as a result, the configuration of regions for that struct is likely to be unique and meaningful [57].

Each of the structural variables is represented in an item plot which may be partitioned according to its presence (score = 2), or absence (score = 1). The item plots may be partitioned in such a way that the regions of space contain profiles with the same score on that item. Subsequently, there are six possible ways to partition these plots (Fig. 2): (i) the *J*-axis ranges from the bottom left hand side of the plot to the top right, (ii) the *L*-axis spreads the profiles from bottom right to the top left hand side of the plot, (iii) horizontal: partitions along the *X* axis, (iv) vertical: partitions along the *Y* axis, (v) *L*-shaped, and; (vi) inverted *L*-shaped (Q type).

Of the six types of partitioning, two divide the solution space in alignment with the *X* and *Y* axes. Behavioral variables operating in this way are particularly interesting because they give the axes that they define the position of profiles (episodes) within the space their primary meaning. These behavioral variables are therefore the two basic types of competitiveness that shape the differences among interaction episodes. A further two types of division align themselves with the diagonal axes, either forming regions along the competitive scale (*J*) or along the qualitative axis (*L*). Behavioral variables dividing the space along the joint axis relate exclusively to the degree of competitiveness in interactions, such that changes in the occurrence of this behavior will be matched by corresponding changes in the degree of competition [57].

Behaviors aligned with the lateral axis influence only the type of competitiveness involved and so influence only the concern or issue: that is the focus of interaction. Finally, a struct may also adopt one of two secondary roles in partitioning the POSAC space, either accentuating (*Q*) or attenuating (*P*) the possibilities to discriminate between points that are high on *X* or *Y*. Behavioral variables that accentuate the space increase the amount of qualitative variation in episodes at higher points on the scale, while attenuating variables increase the homogeneity of interactions such that profiles high on *X* will also be high on *Y*. The interpretation of POSAC output is further illustrated in the literature [66,67].

The above discussion suggests the importance of a preference closeness measure with concern to preference similarity oriented problems and incorporating decision-maker's preferences in cluster analysis. The basic idea is that all the objects inside the same cluster are similar in the sense that they are preferred, indifferent and incomparable to more or less the same objects (similar profiles) [68,69].

There has been a limited attempt in the literature to use POSA software for engineering design. In this paper, the proposed methodology adopted POSA software HUDAP and CoPlot to solve a decision design problem. The objective is to find the best coagulation performance frontiers (min objective function) from a set of jar test alternatives considering both quantitative and qualitative data. The proposed methodology solves the decision design problem in a hierarchical framework, as illustrated in Fig. 3.

2.2. Objective utility function for coagulation/flocculation process

Environmental engineering is concerned with the solid material in a wide range of natural waters and wastewaters and the source of pollution: domestic and industrial wastewater. All organic compounds, except few, can be oxidized by the action of strong oxidizing agents under acid conditions, regardless of the biological assimilability of the substances. This test (COD) is a rapid & precise

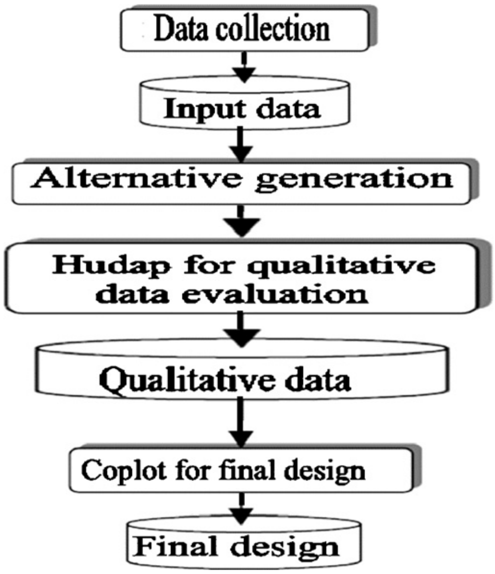

Fig. 3. Proposed methodology.

method: oxidation by potassium dichromate in acid solution and the excess dichromate is titrated with standard ferrous ammonium sulphate using ferroin as an indicator [70]. COD is used for determination of aggregate organic matter and measures organic strength of both domestic and industrial wastewater.

The comparative performance of coagulation/flocculation process by the various alternative methods is very complex and is highly dependable on various site-specific operational and economic parameters. Working to solve this problem, a pilot plant can be added in line to understand the origins of the effluent. In order to support adequate selection the decision-maker has to define a utility function (Z_T). The (objective) utility function is the minimization of the total annual expenses and the minimization of the organic constituent in the treated wastewater.

$$\begin{aligned} Z_T = \text{Min}\{ & (\text{chemical constituant added}) + (\text{chemical cost}) \\ & + (\text{chemical oxygen demand, COD}) \\ & + (\text{total fixed solids, TFS}) + (\text{total volatile solids, TVS}) \\ & + (\text{total suspended solids, TSS})\} \end{aligned} \quad (5)$$

The objective function is subject to a series of specific site constraints and variables (Table 1): (i) treatment expenses (chemical constituent added. Alum is supplied as a solid and a special equipment and operation is needed for Alum solution preparation), (ii) coagulant cost, (iii) return for improved Chemical Oxygen Demand (COD), (iv) return for improved Total Fixed Solids [TFS-residue that remains after sample has been evaporated and dried at 103–105 °C and later ignited at 500 ± 50 °C], (v) return for improved Total Volatile Solids [TVS-solids that volatized and burned off after sample has been evaporated and dried at 103–105 °C and later ignited at 500 ± 50 °C], and; (vi) return for improved Total Suspended Solids [TSS-portion of solids that retained on filter with specified pore size (1.58 μm), measured after sample has been evaporated and dried at 103–105 °C].

Chemical prices can be found at the internet (e.g. ICIS Chemical Business). The posted prices do not necessarily represent level at which transactions may have actually occurred, nor do they represent bid or asked price. The prices are intended as a guide and not to be used as a basis for negotiations between producers and customers. According to ICIS Chemical Business the coagulant prices are: ALUM 331–425 $/ton (100 lb. bags, technical grade, 17% Al_2O_3) and Ferric Chloride 300–351 $/ton (Tanks, technical grade, 100% basis). In this research analysis the coagulants prices were based on database of purchase price in 2009 delivered by "Arrow Ecology": Alum, 253 $/ton (dry) & Ferric Chloride 472 $/ton (dry).

Wastewater raw drainage (and dilution) treated by coagulation/flocculation process is defined by the following chemical characteristics based on Israel National Carrier feed: electrical conductivity 1.0 dS/m, Na^{1+} 60–100 mg/l, Ca^{2+} 45–50 mg/l, Mg^{2+} 20–25 mg/l, Cl^{-1} 200–220 mg/l, $SO_4{}^{-2}$ 17–20 mg/l, $HCO_3{}^{-1}$ 250–300 mg/l [71].

2.3. Analytical methods

Samples were collected from the drainage of a sequencing batch reactor (SBR) plant of the "Arrow Ecology" in Hiria (Israel) and the tests were carried out on the same day. Coagulation and flocculation studies were performed in a standard jar-test apparatus (Velp Scientifica JLT6) comprised of six paddle rotors and equipped with 6 beakers of 1 l volume. JLT6 Flocculators has disconnectable lighted back panel; microprocessor controlled timer with 2 different scales of 0–999 min, 0–99 h or continuous; digital display of speed and time remaining; stainless steel stirring rods adjustable in height by a self locking chuck; individual speed selector for each stirrer via DC gear motor.

Samples were diluted 1:50 with potable water and thoroughly agitated (100 rpm) for re-suspension of settled solids before any tests were conducted. Chemical reagents used as coagulants are commercially available from "Arrow Ecology" company and include Alum [$Al_2(SO_4)_3{\cdot}18H_2O$] and Ferric Chloride ($FeCl_3$).

The initial rapid mixing for all experiments was taken as 5 min (100 rpm) and for slow mixing at 25 min (25 rpm). After settling (duration 30 min), about 50 ml of supernatant was withdrawn using

Table 1
Variables & cost data.

Variable	Alum	Ferric Chloride	Remarks
Cost[a] $/dry ton	253	472	Based on "Arrow Ecology" data and chemical market price in 2009
Treatment expenses	High	Low	Ferric Chloride is sold as solution in containers and Alum is sold as white flakes in Packing PP/PE 50 kg/bag. Alum treatment systems provide Alum solution 10% (w/v). There are no treatment expenses for Ferric Chloride
Chemical added & safety expenses	Low	Medium	Ferric Chloride: ensures ventilation, wear protective equipment to prevent skin and eye contact and inhalation of vapors. Maintain eye wash fountain and quick-drench facilities in work area. Keep containers closed when not in use. Not flammable On burning will emit toxic fumes such as Hydrogen Chloride. Alum: where there is potential for skin contact, wear impermeable gloves. With good ventilation, vapor concentrations will be below exposure limit
COD	–	–	Based on jar test
TFS	–	–	Based on jar test
TVS	–	–	Based on jar test
TSS	–	–	Based on jar test

[a] Technical grade, based on database of purchase prices in 2009 delivered by "Arrow Ecology".

Table 2
Samples treated with Alum.

	Sample	Code	Alum (ppm)	Cent (m^3-feed[a])	COD (ppm)	TSS (ppm)	TFS (ppm)	TVS (ppm)
1	Blank[b]	AL1	0	0	0	0	0	0
2	[c]	AL100	100	2.5	116	820	412	408
3	[c]	AL200	200	5.1	84	692	392	300
4	[c]	AL350	350	8.9	136	880	160	720
5	[c]	AL450	450	11.4	48	872	424	448
6	[c]	AL550	550	13.9	36	848	412	436
7	Raw drainage	AL7	0	0	21,400	29,816	14,140	15,676

[a] 253 $/ton (dry) technical grade, based on database of purchase prices in 2009 delivered by "Arrow Ecology".
The blank sample in is referred to potable water (supplied by the Israel National Water Carrier) that contains no organic constituents.
[b] pH blank = 7.0 ± 0.1.
[c] pH samples = 7.8 ± 0.1.

a plastic syringe from the point located about 4 cm below liquid level for the determination of COD, TSS, TFS, and TVS (Analyses were made in triplicates).

Pure Alum is white and gives a water-white solution. However the presence of iron as an impurity is common which gives the chemical or the solution a yellow or even an orange color. The wastewater plant uses cheap Alum that was prepared from low-grade clays and from waste acid. This leads to the presence of undesirable concentrations of heavy metals in the solution. Metals are not a problem as it is a coagulant in its own right and tends to assist the coagulation process. Ferric Chloride is originally a waste product from spent pickling solution and the wastewater plant uses a cheap product that was prepared from reacting scrap iron with hydrochloric acid and supplied as a solution of about 40% strength as $FeCl_3$.

Coagulant quality control was performed by the measurement of the specific gravity which is both rapid and simple using a hydrometer. If this parameter was differs by more than (±5%) other parameters such as the pH and the viscosity were checked.

Standards of good laboratory practice such as the maintenance and periodic assessment of equipment, instrumentation, consumable supplies were practiced. Precautions were taken to ensure that (i) the workers were familiar with the dangers and treatment associated with these coagulants, and (ii) there were minimum interferences caused by changes in raw water conditions.

Results and experimental observations were recorded in bound notebooks and maintained electronic formats (spreadsheet, instrument record files). Quality assurance and quality control procedures (analysis of method blanks, check samples) were adhered to and records of these measures were kept.

3. Results and discussion

3.1. Jar test results

The characteristics of raw drainage and Alum coagulation/flocculation are presented in Tables 2 and 3. In terms of COD and TSS, there is a high concentration of organic matter. Equal volumes (1000 ml) of measured sample were delivered into each of the jars. Orion Model 420A was used for pH measurements and reagent grade chemical solutions (hydrochloric acid and Sodium hydroxide) were used for controlling the pH of samples.

As reported COD, TSS, TFS and TVS concentrations were specific to Alum test (Table 2) and Ferric Chloride test (Table 3), e.g. the mean values of COD in the effluent varied from 55 mg/l (Ferric Chloride treatment) to 84 mg/l (Alum treatment). COD removal efficiency in both treatments was very high (Ferric Chloride, 99.7% & Alum 99.6%). According to the test results the removal efficiency was almost stable (more than 95%) regarding the organic variables COD, TSS, TFS and TVS. Test result pointed out that coagulation process can guarantee high rejection of organic constituents for wastewater treatment plants.

3.2. Statistical analysis

In order to examine the behavior across all profiles of interaction, the data (Tables 2 and 3) were subjected to a Partial Order Analysis {HUDAP software (MPOSAC)} [72,73]. The MPOSAC algorithm provides a coefficient for the goodness of fit for the representation of the partial order, named CORREP (Eq. (3)) and specifies the proportion of structuple pairs correctly represented by MPOSAC: (i) proportion of comparable pairs correctly represented is 0.9697, (ii) proportion of profile-pairs correctly represented is 1.0000, and; (iii) proportion of incomparable pairs correctly represented is 0.9474.

Item diagram helps the user to detect partitioning of the space into regions according to categories of the specific item. The partitioning may have various directions: with X, Y, J, L, P or Q (Fig. 2). The coefficient of weak monotonicity (Eq. (1)) between each observed item and the partitions along the J, the X and the Y axis (Figs. 1 and 2) is presented in Table 4. One can detect these directions in Table 4: a high correlation indicates a common direction. So, item TFS goes with Y (0.98), while COD goes with X (0.97). This means that the partly ordered space is essentially spanned by these two variables.

Table 3
Samples treated with Ferric Chloride.

	Sample	Code	Ferric (ppm)	Cent (m^3-feed[a])	COD (ppm)	TSS (ppm)	TFS (ppm)	TVS (ppm)
1	Blank[b]	FC1	0	0	0	0	0	0
2	[c]	FC100	100	4.7	20	564	116	448
3	[c]	FC200	200	9.4	40	960	388	572
4	[c]	FC350	350	16.5	120	944	268	676
5	[c]	FC450	450	21.2	44	1204	536	668
6	[c]	FC550	550	26.0	52	1096	268	828
7	Raw drainage	FC7	0	0	21,400	29,816	14,140	15,676

[a] 472 $/ton (dry) technical grade, based on database of purchase prices in 2009 delivered by "Arrow Ecology".
The blank sample referred to potable water (supplied by the Israel National Water Carrier) that contains no organic constituents.
[b] pH blank = 7.0 ± 0.1.
[c] pH samples = 6.5 ± 0.1.

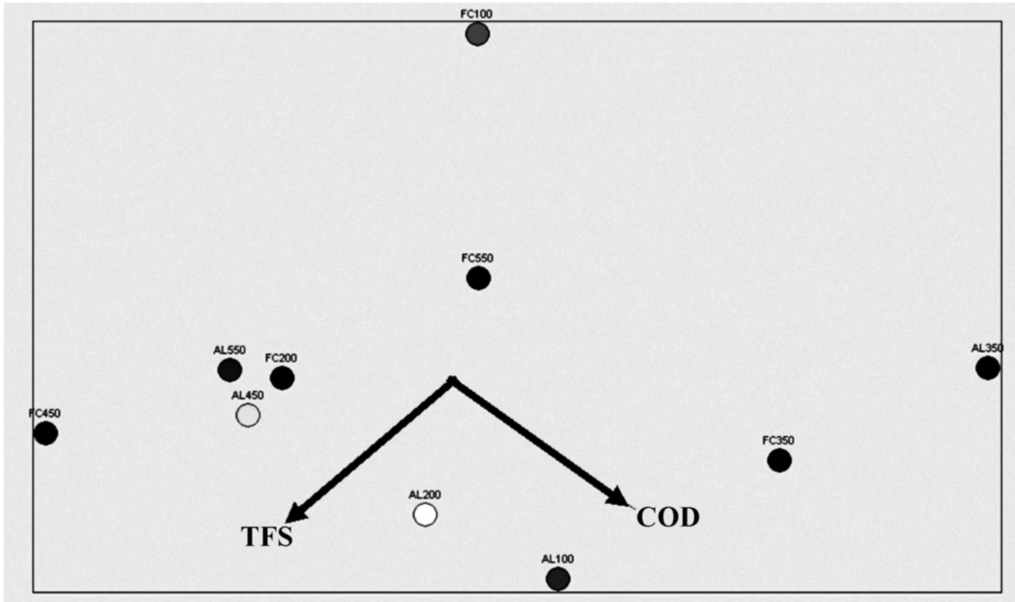

Fig. 4. Map, generated by the CoPlot software.

Table 4
Coefficient of weak monotonicity between each observed items and the partitions along the *J*, the *X* and the *Y* axis[a].

Item name	*J*	*X*	*Y*
PPM	0.85	−0.07	0.65
Cent (m^3)	0.85	0.22	0.37
COD	0.91	**0.97**	−0.44
TSS	0.92	0.51	0.25
TFS	0.91	−0.47	**0.98**
TVS	0.52	0.70	−0.51

[a] High correlation (bold numbers) indicates a common direction.

The statistical analysis shows the power of POSA to organize objects simultaneously according to level of the sorted criteria and as functions of the content meaning of the base coordinates *X* and *Y*.

The COD and TFS variables (concerning Tables 2 and 3) were structured by the CoPlot software and the output is displayed in Fig. 4. The Coefficient of Alienation (Eq. (2)) equals 0.056. COD axis degree is −44 and TFS axis degree is −130 (nearly orthogonal axis). COD and TFS axis have the same Pearson correlation (0.99).

Overall, based on utility function and jar tests, HUDAP and CoPlot software, 100 ppm Ferric Chloride (Code FC100) is the best system among all systems studied which shows the highest efficiency in terms of economic aspects and reduction of COD, TSS and TFS.

4. Conclusions

The design of a cost effective wastewater treatment process to achieve a desired good quality for irrigation can be very difficult, as a large number of treatment options are available. This process is further compounded by the many criteria that are needed to be considered in the selection course of action. A utility function was developed in order to test the performance of two commonly used coagulation–flocculation aids (Alum and Ferric Chloride) for the treatment of the drainage of a sequencing batch reactor (SBR) plant of the “Arrow Ecology” in Hiria (Israel).

In order to solve the utility function, an expert systems software based on partial order methodology were used. This study demonstrates the possibilities and appropriateness of using POSA for selection of the optimal coagulant and provides a systematical decision making framework with several characteristics: (i) the importance of different performances of treatment systems can be evaluated using multiple criteria – both quantitative and qualitative – rather than profitability alone, (ii) the use of ratings makes it possible to evaluate the applicability of different alternatives for the end user, (iii) the use of POSA method provides an effective way of documenting the management process, and; (iv) the proposed approach forms the basis for a continuous process of planning and managing technology selection, so that the priorities of the treatment processes can easily be modified and updated.

Overall, based on jar tests and POSA models, 100 ppm Ferric Chloride is the best system among all systems studied which shows the highest efficiency in terms of reduction of COD, TSS, TVS and TFS. This study evidenced once again that coagulation process can assure the limits of organics for municipal wastewater treatment plants providing high removal efficiency using relatively low level of Alum or Ferric Chloride if the process is well optimized and operated.

The work presents the variability effects of process variables and shows how POSA technique points out the importance of each criterion. This characteristic is of direct scientific and engineering concern and provides useful explanations for analysis of coagulation/flocculation performance and selection.

It is highly recommended that plant trials be conducted whenever possible, since the jar test is not absolutely infallible and a true assessment of the coagulant is only possible at plant scale. Plant trials should be conducted for a minimum of three weeks and preferably six weeks for each coagulant in order to obtain a true reflection of the performance of each chemical. Periods shorter than this may not be long enough to allow complete eradication of the previously used coagulant from the floc blanket.

Once a coagulant has been selected, it is important to ensure that the quality of the chemical remains constant between deliveries. Jar tests can be used for this purpose, but simpler, more rapid tests can be used to determine whether certain important parameters are remaining consistent between deliveries. In the case of inorganic coagulants such as Alum or Ferric Chloride, the measurement of specific gravity can be carried out.

Another test that can be used on-site to monitor plant performance is known as the Cascade test. This is similar to the jar test, except that the dosing of treatment chemicals takes place on the plant. Samples are then collected of the dosed water prior to entering the clarifiers and flocculation and settling are simulated using the jar test apparatus. These tests allows the plant operator a rapid means of assessing the impact of various changes in treatment chemical type and dose on plant performance without waiting for the full effects of a dosage change to pass through the works.

Acknowledgements

The authors wish to thank Prof. Cohen Erik (Bar-Ilan University) and Dr. Amar Reuven (Hebrew University of Jerusalem) for useful comments that allowed the improvement of the article, especially for drawing our attention to helpful discussions and for their valuable comments and recommendations concerning HUDAP software methodology. The authors would like to thank the anonymous referees and the editor for their helpful comments on an earlier version of this manuscript, which helped improve the clarity and exposition of the current paper.

References

[1] J.M. Ebeling, P.I. Sibrell, S.R. Ogden, S.T. Summerfelt, Evaluation of chemical coagulation–flocculation aids for the removal of suspended solids and phosphorus from intensive recirculating aquaculture effluent discharge, Aquacult. Eng. 29 (2003) 23–42.

[2] M.I. Aguilar, J. Saez, M. Llorens, A. Soler, J.F. Ortuno, Nutrient removal and sludge production in the coagulation–flocculation process, Water Res. 36 (2002) 2910–2919.

[3] M.I. Aguilar, J. Saez, M. Llorens, A. Soler, J.F. Ortuno, Microscopic observation of particle reduction in slaughterhouse wastewater by coagulation–flocculation using ferric sulphate as coagulant and different coagulant aids, Water Res. 37 (2003) 2233–2241.

[4] F. Friedler, I. Katz, C.G. Dosoretz, Chlorination and coagulation as pretreatments for greywater desalination, Desalination 222 (2008) 38–49.

[5] J. Duan, J. Gregory, Coagulation by hydrolysing metal salts, Adv. Colloid Interf. 100–102 (2003) 475–502.

[6] N.Z. Al-Mutairi, M.F. Hamoda, I. Al-Ghusain, Coagulant selection and sludge conditioning in a slaughterhouse wastewater treatment plant, Bioresour. Technol. 95 (2004) 115–119.

[7] H. Selçuk, D. Kaptan, S. Meriç, Coagulation of textile wastewater using alum and Fe(III) salts, Fresenius Environ. Bull. 13 (10) (2004) 1045–1048.

[8] J.R. Dominguez, J.B. de Heredia, T. Gonzalez, F. Sanchez-Lavado, Evaluation of ferric chloride as a coagulant for cork processing wastewaters. Influence of the operation conditions on the removal of organic matter and settleability parameters, Ind. Eng. Chem. Res. 44 (2005) 6539–6548.

[9] A.V. Jung, V. Chanudet, J. Ghanbaja, B.S. Lartiges, J.L. Bersillon, Coagulation of humic substances and dissolved organic matter with a ferric salt: an electron energy loss spectroscopy investigation, Water Res. 39 (2005) 3849–3862.

[10] A. Matilainen, N. Lindqvist, T. Tuhkanen, Comparison of the efficiency of aluminium and ferric sulphate in the removal of natural organic matter during drinking water treatment process, Environ. Technol. 26 (2005) 867–875.

[11] S.G.S. Rodríguez, M.D. Kennedy, A. Diepeveen, H. Prummel, J.C. Schippers, Optimization of PACl dose to reduce RO cleaning in an IMS, Desalination 220 (2008) 239–251.

[12] M. Yan, D. Wang, J. Qu, J. Ni, C.W.K. Chow, Enhanced coagulation for high alkalinity and micro-polluted water: the third way through coagulant optimization, Water Res. 42 (2008) 2278–2286.

[13] A.G.M. Silva, M.O. Hornes, M.L. Mitterer, M.I. Queiroz, Application of coagulants in pretreatment of fish wastewater using factorial design, Desalination Water Treat. 1 (2009) 208–214.

[14] A.A. Tatsi, A.I. Zouboulis, K.A. Matis, P. Samaras, Coagulation–flocculation pretreatment of sanitary landfill leachates, Chemosphere 53 (2003) 737–744.

[15] S.H. Kim, B.H. Moon, H.L. Lee, Effects of pH and dosage on pollutant removal and floc structure during coagulation, Microchem. J. 68 (2001) 197–203.

[16] M. Franceschia, A. Girou, A.M. Carro-Diaz, M.T. Maurette, Optimisation of the coagulation– flocculation process of raw water by optimal design method, Water Res. 36 (2002) 3561–3572.

[17] M. Guida, M. Mattei, C.D. Rocca, G. Melluso, S. Meriç, Optimization of alum-coagulation/flocculation for COD and TSS removal from five municipal wastewaters, Desalination 211 (2007) 113–127.

[18] F. Xiao, J. Ma, P. Yi, J.C.H. Huang, Effects of low temperature on coagulation of kaolinite Suspensions, Water Res. 42 (2008) 2983–2992.

[19] J.R. Brunet, G.D. Garson, Dimensionality analysis as a computerized tool for strategic planning in policing and security, Soc. Sci. Comput. Rev. 27 (2) (2009) 228–242.

[20] M. Lavine, Mathematical techniques and the number of groups, Behav. Brain Sci. 31 (1) (2008) 83–84.

[21] K. Voigt, R. Brueggemann, H. Scherb, H. Shen, K.W. Schramm, Evaluating the relationship between chemical exposure and cryptorchidism, Environ. Modell. Softw. 25 (12) (2010) 1801–1812.

[22] L. Carlsen, Giving molecules an identity. On the interplay between QSARs and partial order ranking, Molecules 9 (2004) 1010–1018.

[23] L. Carlsen, Hierarchical partial order ranking, Environ. Pollut. 155 (2008) 247–253.

[24] R. Brüggemann, K. Voigt, G. Restrepo, U. Simon, The concept of stability fields and hot spots in ranking of environmental chemicals, Environ. Modell. Softw. 23 (2008) 1000–1012.

[25] A.K. Gautam, S. Kumar, P.C. Sabumon, Preliminary study of physico-chemical treatment options for hospital wastewater, J. Environ. Manage. 83 (2007) 298–306.

[26] S.A.R. Ahmed, Fast-track evaluation of a compact chemically enhanced-trickling filter system, Braz. J. Chem. Eng. 24 (2) (2007) 171–184.

[27] S. Bhatia, Z. Othman, A.L. Ahmad, Coagulation–flocculation process for POME treatment using *Moringa oleifera* seeds extract: optimization studies, Chem. Eng. J. 133 (2007) 205–212.

[28] K. Park, Y.T. Seo, G.D. Whang, W. Lee, Optimizing enhanced coagulation for DOC removal with ultrafiltration membrane separation using response surface: methods and particle trajectory analysis, Water Sci. Technol. 42 (2000) 187–192.

[29] A.L. Ahmad, S. Ismail, S. Bhatia, Optimization of coagulation–flocculation process for palm oil mill effluent using response surface methodology, Environ. Sci. Technol. 39 (2005) 2828–2834.

[30] A.L. Ahmad, S.S. Wong, T.T. Teng, A. Zuhairi, Optimization of coagulation–flocculation process for pulp and paper mill effluent by response surface methodological analysis, J. Hazard. Mater. 145 (2007) 162–168.

[31] J.P. Wang, Y.Z. Chen, X.W. Ge, H.Q. Yu, Optimization of coagulation–flocculation process for a paper-recycling wastewater treatment using response surface methodology, Colloids Surf. A: Physicochem. Eng. Aspects 302 (2007) 204–210.

[32] S.S. Moghaddam, M.R.A. Moghaddam, M.A. Rami, Coagulation/flocculation process for dye removal using sludge from water treatment plant: optimization through response surface methodology, J. Hazard. Mater. 175 (2010) 651–657.

[33] H.R. Maier, N. Morgan, K. Chow, Artificial neural networks for predicting optimal alum doses and treated water quality parameters, Environ. Modell. Softw. 19 (2004) 485–494.

[34] H. Bae, S. Kim, Y.J. Kim, Decision algorithm based on data mining for coagulant type and dosage in water treatment systems, Water Sci. Technol. 53 (4–5) (2006) 321–329.

[35] C.L. Chen, P.L. Hou, Fuzzy model identification and control system design for coagulation chemical dosing of potable water, Water Sci. Technol.: Water Supply 6 (3) (2006) 97–104.

[36] M. Al-Abri, N. Hilal, Artificial neural network simulation of combined humic substance coagulation and membrane filtration, Chem. Eng. J. 141 (2008) 27–34.

[37] P. Annoni, R. Brüggemann, The dualistic approach of FCA: a further insight into Ontario Lake sediments, Chemosphere 70 (2008) 2025–2031.

[38] D. Lerche, R. Brüggemann, P. Sorensen, D. Nielsen, A comparison of partial order technique with three methods of multi-criteria analysis for ranking of chemical substances, J. Chem. Inf. Sci. 42 (2002) 1086–1098.

[39] R. Brüggemann, D. Lerche, P.B. Sørensen, L. Carlsen, Estimation of average ranks by a local partial order model, J. Chem. Inf. Comp. Sci. 44 (2004) 618–625.

[40] G.P. Patil, C. Taillie, Multiple indicators, partially ordered sets, and linear extensions: multi-criterion ranking and prioritization, Environ. Ecol. Stat. 11 (2004) 199–228.

[41] K. Voigt, G. Welzl, R. Brüggemann, Data analysis of environmental air pollutant monitoring systems in Europe, Environmetrics 15 (2004) 577–596.

[42] R. Brüggemann, L. Carlsen, Introduction to partial order theory exemplified by the evaluation of sampling sites, in: R. Brüggemann, L. Carlsen (Eds.), Partial Order in Environmental Sciences and Chemistry, Springer, Berlin, 2006, pp. 61–110.

[43] P.L. Hammer, A. Kogan, M.M. Lejeune, Computing, artificial intelligence and information management modeling country risk ratings using partial orders, Eur. J. Oper. Res. 175 (2006) 836–859.

[44] A. Ünlüsoy, M. de Haan, P.M. Leseman, C. van Kruistum, Gender differences in adolescents' out-of-school literacy practices: a multifaceted approach, Comput. Educ. 55 (2) (2010) 742–751.

[45] S. Pudenz, R. Brüggemann, B. Luther, A. Kaune, K. Kreimes, An algebraic/graphical tool to compare ecosystems with respect to their pollution v: cluster analysis and Hasse diagrams, Chemosphere 40 (12) (2000) 1373–1382.

[46] R. Brüggemann, E. Halfon, G. Welzl, K. Voigt, C. Steinberg, Applying the concept of partially ordered sets on the ranking of near-shore sediments by a battery of tests, J. Chem. Inf. Comp. Sci. 41 (2001) 918–925.

[47] H. Hollert, S. Heise, S. Pudenz, R. Brüggemann, W. Ahlf, T. Brambeck, Application of sediment quality triad and different statistical approaches (Hasse diagrams and fuzzy logic) for the comparative evaluation of small streams, Ecotoxicology 11 (5) (2002) 311–321.

[48] K. Voigt, R. Brüggemann, S. Pudenz, A multi-criteria evaluation of environmental databases using Hasse diagram technique (ProRank) software, Environ. Modell. Softw. 21 (2006) 1587–1597.

[49] R. Brüggemann, G. Welzl, K. Voigt, Order theoretical tools for the evaluation of complex regional pollution patterns, J. Chem. Inf. Comp. Sci. 43 (2003) 1771–1779.

[50] P.B. Sørensen, R. Brüggemann, L. Carlsen, B.B. Mogensen, J. Kreuger, S. Pudenz, Analysis of monitoring data of pesticide residues in surface waters using partial order ranking theory, Environ. Toxicol. Chem. 22 (3) (2003) 661–670.
[51] G. Restrepo, R. Brüggemann, K. Voigt, Partially ordered sets in the analysis of alkanes fate in rivers, Croat. Chem. Acta 80 (2007) 261–270.
[52] S. Kardaetz, T. Strube, R. Brüggemann, G. Nutzmann, Ecological scenarios analyzed and evaluated by a shallow lake model, J. Environ. Manage. 88 (2008) 120–135.
[53] G. Restrepo, M. Weckert, R. Brüggemann, S. Gerstmann, H. Frank, Ranking of refrigerants, Environ. Sci. Technol. 42 (2008) 2925–2930.
[54] A. Bhattacharjee, Partial orders with respect to continuous covariates and tests for the proportional hazards model, J. Stat. Plan. Infer. 141 (1) (2011) 243–261.
[55] B. Luther, R. Brüggemann, S. Pudenz, An Approach to combine cluster analysis with order theoretical tools in problems of environmental pollution, Match-Commun. Math. Ch. 42 (2000) 119–143.
[56] M. Pavan, R. Todeschini, New indices for analysing partial ranking diagrams, Anal. Chim. Acta 515 (2004) 167–181.
[57] S. Shye, Multiple Scaling: The Theory and Application of Partial Order Scalogram Analysis, Amsterdam, North Holland, 1995.
[58] S. Pudenz, ProRank – Software for partial ordering, Match-Commun. Math. Ch. 54 (3) (2005) 611–622.
[59] L. Carlsen, R. Brüggemann, Accumulating partial order ranking, Environ. Modell. Softw. 23 (2008) 986–993.
[60] P. Annoni, Different ranking methods: potentialities and pitfalls for the case of European opinion poll, Environ. Ecol. Stat. 14 (4) (2007) 453–471.
[61] U. Simon, R. Brüggemann, S. Mey, S. Pudenz, METEOR – application of a decision support tool based on discrete mathematics, Match-Commun. Math. Ch. 54 (3) (2005) 623–642.
[62] U. Simon, R. Brüggemann, H. Behrendt, E. Shulenberger, S. Pudenz, METEOR: a step-by-step procedure to explore effects of indicator aggregation in multi criteria decision aiding – application to water management in Berlin, Germany, Acta Hydroch. Hydrob. 34 (2006) 126–136.
[63] J. Hornik, E.H. Cohen, R. Amar, A Facet metatheoretical approach to advance consumer behavior knowledge, Psychol. Marketing 241 (9) (2007) 787–813.
[64] A. Raveh, CoPlot: a graphic display method for geometrical representations of MCDM, Eur. J. Oper. Res. 125 (2000) 670–678.
[65] D.M. Bravata, K.G. Shojania, I. Olkin, A. Raveh, CoPlot: a tool for visualizing multivariate data in medicine, Statist. Med. 27 (2008) 2234–2247.
[66] R. Brüggemann, K. Voigt, S. Pudenz, Data handling in science and technology: partial ordering and Hasse diagrams, Appl. Chem. Softw. 27 (3) (2008) 73–95.
[67] L.E. Porter, L.J. Alison, A partially ordered scale of influence in violent group behavior: an example from gang rape, Small Group Res. 32 (2001) 475–497.
[68] R. Belohlavek, V. Vychodil, Discovery of optimal factors in binary data via a novel method of matrix decomposition, J. Comput. Syst. Sci. 76 (2010) 3–20.
[69] E. Fernandez, J. Navarro, S. Bernal, Handling multicriteria preferences in cluster analysis, Eur. J. Oper. Res. 202 (2010) 819–827.
[70] K.P. Singh, A. Malik, D. Mohan, S. Sinha, V.K. Singh, Chemometric data analysis of pollutants in wastewater—a case study, Anal. Chim. Acta 532 (1) (2005) 15–25.
[71] U. Yermiyahu, A. Tal, A. Ben-Gal, A. Bar-Tal, J. Tarchitzky, O. Lahav, Rethinking desalinated water quality and agriculture, Environ. Sci. 318 (2007) 920–921.
[72] R. Amar, Mathematical formulation of regionality in SSA and POSAC/MPOSAC, in: D. Elizur (Ed.), Facet Theory, University Karlovy V, Prague, 2001, pp. 63–74.
[73] R. Amar, S. Toledano, HUDAP Manual, The Hebrew University, Jerusalem, 2001.